

EXHIBIT 3

Regression Analysis and Discounts for Lack of Marketability

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This article develops a multivariate regression model to identify factors that impact the discounts observed in the private placement of publicly traded stock. The predicted private placement discounts derived from the model can be used as a basis for determining discounts for lack of marketability for ownership interests in closely held companies or illiquid blocks of publicly traded stock. The regression model features a combination of transaction-specific and company-specific data and also takes into account contemporaneous market conditions. The results from the model thus reflect both unique attributes of each company as well as the market environment on the valuation date.

Introduction

Private placements of publicly traded stock are undertaken for a variety of reasons, ranging from funding a specific project, such as building a new factory or funding a clinical trial, to paying off debt or funding day-to-day operations.

In this article we discuss the development of a multivariate regression model, built on private placement data, that can be directly employed to empirically address the issue of discounts for lack of marketability (DLOMs)¹ associated with closely held securities. In the pages that follow, we will discuss the origin of our regression model, observed data from our research, and previous regression models. We will then address the specifics of the regression model. We will address how predicted private placement discounts determined from our regression analysis are used in determining the DLOM. Finally, we will provide a case study.

A primary objective of our current study was to utilize current transactional data and contemporaneous financial and market data to devise client and time-specific DLOMs. To address the issues, we set out to gather the valuation profession's most comprehensive data set that included every private placement of publicly traded stock in two major databases² from 2000 to September 2009, as well as roughly 220 transactions from the period 1980 to 1999 that were part of an original proprietary restricted stock study conducted by Management Planning, Inc., several

years ago. Our data set currently contains 1,863 transactions, after excluding companies whose closing price was less than \$1 per share the day before the announcement.³

For each transaction we collected a variety of data, including the following: the transaction date; the price of the publicly traded stock of the private placement issuer on the date of the transaction (as well as the day before)⁴; the private placement price; the status of registration and/or registration rights of the privately placed shares; the block size; and the holding period restrictions that were in place at the time of the transaction. We also gathered a range of issuer-specific data including, but not limited to, market capitalization, historical stock price volatility, operating performance metrics, and other measures of financial strength. Since we also hypothesized that contemporaneous market conditions affect the private placement discounts required by investors on restricted securities, we compiled data on broad financial market conditions, including stock market indices and interest rates.

We classified each private transaction based on registration status (registered, unregistered, registration rights, or agreement to register) at two time intervals: first, its registration status at the time the transaction was announced and second, whether its registration status changed in the ensuing one-year period (six months for transactions that occurred after February 2008).

The Data

As we mentioned, our data set includes 1,863 observations. Our first cut of the data was to look at the transactions in the context of their registration status. As Table 1

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Table 1
Discounts by Registration Status

Registration Status	Observations, No.	Mean Discount, %*
Unregistered	402	22.1
Registered	203	8.7
Registration Rights, Agreement to Register, or Subsequently Registered†	1,258	15.0
Total	1,863	15.9

* Discount (from the private placement price) measured using the closing price the day before the private placement was announced.

† Of the 1,258 transactions, 1,164 were later registered, and the remaining 94 were not registered.

illustrates, 402 transactions were unregistered and 203 were registered. The remaining 1,258 transactions either had registration rights or agreements to register at a later date or were subsequently registered.

Table 1 summarizes the registration status of the private placement shares at the time of the announcement of the transaction and whether that status changed within a one-year period (or within a six-month period for transactions that occurred after 15 February 2008, when the SEC-mandated holding period was shortened to six months) after the announcement.

The overall average discount for these 1,863 private placements was 15.9%. For the 203 companies whose privately placed stock was registered at the time of the transaction, the average discount was 8.7%. For the 402 companies whose stock was unregistered at the time of the private placement and who were not registered within the following year, the average discount was 22.1%.

Transactions in unregistered stock can be grouped into four distinct time periods that match the changes in holding period restrictions, as laid out in SEC Rule 144⁵: (i) those occurring prior to 1990 (time period I); (ii) those occurring between 1990 and April 1997 (time period II); (iii) those occurring between April 1997 and February 2008 (time period III); and (iv) those occurring after February 2008 (time period IV).

Table 2
Discounts by Time Period

Time Period	Covering	Number of Observations	Mean Discount, %	Median Discount, %
I	Pre-1990	79	30.5	32.3
II	1990 to April 1997	110	25.1	22.5
III	May 1997 to February 2008	164	20.8	16.6
IV	February 2008 to December 2008	49	5.9	5.0
	Total	402	22.1	19.6

Table 2 shows the mean and median discounts for the 402 transactions involving unregistered shares during each of the four time periods in our study. Table 2 shows that discounts involving private placements have fallen over the four periods described. The mean discount has fallen from 30.5% in time period I to 25.1% in time period II, 20.8% in time period III, and 5.9% in time period IV.

As noted earlier, we classified each private transaction by registration status (registered, unregistered, registration rights, agreement to register) at two time intervals: first, at the time the transaction was announced and second, at the one-year period (or the six-month period for transactions in time period IV). Thus, while a company's private placement may have involved shares that were unregistered at the time of the announcement of the sale, we reviewed SEC filings for that company for the following year (or six months) to see if the shares sold in that placement were later registered. One might reasonably expect that a private placement investor who knew that his shares, although unregistered at the time of the purchase, could (or would) later be registered would accept a smaller private placement discount as a result of the impending registration, as shown in Table 3.

Table 3 summarizes the registration status of the private placement shares at the time of the announcement of the transaction and whether that status changed within a one-year period after the announcement. Referring back to Table 1, of the 1,258 transactions identified as having some nexus to subsequent registration, 297 were actually registered within a year of the placement date.

Shares that were registered within one year of the placement (or six months in time period IV) were placed

Table 3
Discounts for Saares that were Unregistered at Announcement

Unregistered at Announcement– Registration Status One Year Later	Number of Observations	Mean Discount, %
Unregistered*	402	22.1
Registered	297	14.4

* These are the same 402 transactions listed in Table 2.

Table 4
 Summary Statistics for All Placements

	Mean	Median	Minimum	Maximum
Discount/(Premium), %	15.9	13.3	-88.9	82.2
Size of Placement, \$MM	24.6	8.8	0.00825	5,600.0
Market Capitalization, \$MM	277.6	83.8	2.2	49,268.0
Company Volatility, %	95.6	82.2	8.6	790.9
S&P 500 Twelve-Month Return, %	4.2	8.0	-47.7	59.1
S&P 500 Volatility, %	17.0	14.3	8.1	43.1
Thirty-Year Treasury Yield, %	5.3	4.9	2.6	15.0
Block to Shares Outstanding, %	18.6	13.2	0.01	412.1

at a discount of 14.4%, compared to a 22.1% discount for those shares that remained unregistered one year after the announcement. The difference in the mean discounts was statistically significant at better than the 1% significance level.⁶ This supports the hypothesis that a private placement investor who knew that his shares, although unregistered at the time of the purchase, could (or would) later be registered (within one year) would accept a smaller private placement discount as a result of the impending registration.

Returning to the complete data set, we show summary statistics for several key variables in Table 4.⁷ Notably, the average discount for our complete data set was 15.9% and the median was 13.3%, with a range of a discount of 82.2% to a premium of 88.9%. This wide range was similar to the ranges seen in certain earlier studies (discussed later in this article). Much as earlier authors noted, there are idiosyncratic factors at work here that lead to this high level of variability, such as the investor being promised representation on the board and special relationships between the investor and the company. The average size of the placement was \$24.6 million, with a range from below \$1.0 million to \$5.6 billion.⁸

We expect that there is a more systematic relationship between the independent variables we will discuss and the observed discounts. Unlike the earlier studies, which

we will address shortly, we believe contemporaneous market conditions play a major explanatory role in understanding private placement discounts. As shown in Table 4, we looked at several market condition-related factors, such as the twelve-month return on the S&P 500, the yield on the thirty-year Treasury bond, and the volatility of the S&P 500.

In Table 5, we focus our attention on those private placements that were unregistered at the time of the placement and were not later registered. Again, we believe that these transactions offer the most pure comparison between the two types of stock: those that are readily marketable and those that are restricted. The median discount for these 402 transactions was 19.6%, as shown in Table 2. The median size of the placement was \$4.3 million, and the median market capitalization was just over \$55 million; both values are smaller than the observed medians in Table 4. We believe that these unregistered shares represent the best proxy for identifying the discounts required by purchasers of shares with limited marketability and limited liquidity. We note the materially higher mean (22.1% versus 15.9% for all 1,863 private placements) and median discounts (19.6% versus 13.3% for all) for these 402 transactions with unregistered shares.

Table 5
 Summary Statistics for Unregistered Placements

	Mean	Median	Minimum	Maximum
Discount/(Premium), %	22.1	19.6	-82.2	85.8
Size of Placement, \$MM	3.7	4.3	0.1	5,600.0
Market Capitalization, \$MM	529.4	55.3	2.2	49,268.0
Company Volatility, %	91.4	71.1	13.2	623.2
S&P 500 Twelve-Month Return, %	8.4	9.6	-43.1	59.1
S&P 500 Volatility, %	14.8	12.2	8.1	42.4
Thirty-Year Treasury Yield, %	6.5	5.9	3.0	15.0
Block to Shares Outstanding, %	17.3	10.9	0.4	245.0

Table 6
Summary Statistics by Time Period (Median)

	Overall	Period I (1980 to 1990)	Period II (1990 to April 97)	Period III (April 1997 to February 2008)	Period IV (February 2008 to September 2009)
<i>N</i>	402	79	110	164	49
Discount	19.6	32.3	22.5	16.6	5.0
Size of Placement, \$MM	4.3	3.0	4.7	4.4	5.0
Market Capitalization, \$MM	55.3	31.3	46.2	68.4	114.3
Company Volatility, %	71.1	53.8	68.4	83.8	89.0
S&P 500 Twelve-Month Return, %	9.6	22.5	9.9	9.7	-15.1
S&P 500 Volatility, %	12.2	14.0	9.4	11.4	20.0
Thirty-Year Treasury Yield, %	5.9	10.4	6.9	4.9	4.5
Block to Shares Outstanding, %	10.9	13.4	15.8	9.6	5.5

Table 6 (shown above) traces the evolution of the key variables over the four time periods described earlier. As previously noted, the average private placement discount has fallen over time, due primarily to a loosening of holding period rules, as laid out in SEC Rule 144.

- The median size of the private placement has risen slightly, from \$3.0 million in period I to \$5.0 million in period IV.
- The median market capitalization has risen from \$31.3 million in period I to approximately \$114.3 million in period IV.
- Company volatility has risen from a level of roughly 54% in period I to 68% in period II and to a level of roughly 84% in period III and 89% in period IV.
- The S&P 500 twelve-month return was in excess of 20% per year in period I, approximately 10% in periods II and III, and showed a decline of roughly 15% in period IV, as a result of the financial crisis.
- S&P 500 volatility was 14% per year in period I, declined to a range of 9% to 11% in periods II and III, and has risen to an average level of nearly 20% in period IV (also due primarily to the financial crisis of 2008–2009).
- Thirty-year Treasury yields have declined from 10.4% in period I to 4.5% in period IV.
- Finally, the percent of shares placed, as measured by the ratio of the number of shares placed to the number of shares outstanding, has declined as well, from a level of roughly 13% in period I to 5.5% in period IV.

Table 7 shows a series of tests of the means of the variables by time period. Table 7 shows that for the following variables the mean values varied by time period in a statistically significant manner over time: company volatility, twelve-month change in the S&P 500, S&P 500 volatility, and thirty-year Treasury yield. The following

variables had means that were statistically significantly different in some, but not all, time periods: private placement discount, market capitalization, and size of placement. For quarters to sell under the 1% limit rule, the means were not statistically significantly different in any time period.

In order to properly account for the individual (and simultaneous) effects of the aforementioned variables we developed a multivariate regression model. Before turning to our regression model, we briefly discuss previous regression studies.

Previous Regression Studies

Several regression-based studies have been done in the past several decades that look to identify determinants of private placement discounts, notably those of Wruck,⁹ Hertz and Smith,¹⁰ Silber,¹¹ and Bajaj et al.¹² Wruck had data on 128 private sales of equity from July 1979 through December 1985. The Hertz and Smith study looked at 106 private placement transactions from 1980 to 1987, while that of Silber looked at 69 private placement transactions from 1981 to 1988. Bajaj looked at 88 private placement transactions. Each of these studies found that private placements of unregistered shares, on average, took place at discounts from the publicly traded price.

Many of us in the valuation profession are very familiar with the work of Wruck, Hertz and Smith, Silber, and Bajaj. Criticisms of these studies included issues regarding classification of registered versus unregistered shares and timing of the price used for the private placement. One excellent treatment of these studies was provided by Mitchell and Norwalk¹³ in 2008. We also note that none of these studies included variables for market conditions.

Our goal is to create a more comprehensive model of private placement discounts that is not subject to the criticisms of the earlier studies. In that respect, we offer

Table 7
 Test of Means by Time Period

Variable	Time Period			
	I	II	III	IV
Discount/(Premium)				
Mean Outside of Period, %	15.08	15.14	15.78	16.70
Mean In Period, %	30.48	25.06	15.56	9.55
<i>t</i> -statistic	5.5417***	5.323***	0.193	-5.165***
Size of Placement, \$000				
Mean Outside of Period	21,100	20,800	13,700	21,700
Mean In Period	5,953	16,700	22,700	14,600
<i>t</i> -Statistic	4.743***	0.638	-2.25**	1.908*
Market Capitalization, \$MM				
Mean Outside of Period	236.5	232.3	164.9	239.5
Mean In Period	60.1	193.0	250.4	179.7
<i>t</i> -Statistic	4.856***	0.425	-1.728*	1.275
Company Volatility				
Mean Outside of Period, %	107.0	106.9	100.5	103.5
Mean In Period, %	61.1	77.5	107.0	116.7
<i>t</i> -Statistic	7.356***	8.033***	-2.001**	3.6326***
S&P 500 Twelve-Month Change				
Mean Outside of Period, %	2.4	2.5	-9.0	7.5
Mean In Period, %	19.1	13.1	6.7	-22.3
<i>t</i> -Statistic	-8.641***	9.7849***	-15.206***	39.03***
S&P 500 Volatility				
Mean Outside of Period, %	17.5	17.8	24.6	15.0
Mean In Period, %	15.5	10.2	15.2	31.1
<i>t</i> -Statistic	3.077***	27.200***	17.920***	-33.763***
Thirty-Year Treasury Yield				
Mean Outside of Period	4.99	5.07	5.70	5.33
Mean In Period	10.15	7.14	5.00	4.24
<i>t</i> -Statistic	-23.856***	-29.451***	6.857***	30.356***
Quarters to Sell Under 1% Limit Rule				
Mean Outside of Period	21.6	21.6	19.8	21.8
Mean In Period	20.4	21.4	21.9	18.4
<i>t</i> -Statistic	0.489	0.060	-1.152	1.333

* Statistically significant at the 10% level.

** Statistically significant at the 5% level.

*** Statistically significant at the 1% level.

an analysis based on a far larger data set (greater than 1,860 observations), with discounts measured from the day before the announcement of the private placement¹⁴ and registration status of the shares placed documented in detail. We also demonstrate that contemporaneous market conditions affect private placement discounts (earlier studies do not include these market factors).

The Regression Model

As noted earlier, we believe that private placement discounts are a function of company-specific factors, transaction-specific factors, and contemporaneous market conditions. In this section, we will further discuss these factors and structure a regression model to estimate the relationship between private placement discounts and these factors.

The company-specific variables we analyzed include stock price volatility, market capitalization, and a variety of financial indicators, such as revenue, earnings, leverage, and price-to-book ratios. Our analysis indicated that the relationship between market capitalization and private placement discounts varied over time. Thus, we created an interaction term that had a separate component for each time period. Specifically, we created a variable that was the natural log¹⁵ of market capitalization in time period I that was equal to the natural log of market capitalization if the transaction took place in time period I and was equal to zero otherwise. We created interaction terms for the natural log of market capitalization in time periods II and III as well, with time period IV as the base case. Thus, the interaction term is equal to log market cap in the period in which the transaction occurs and is zero

otherwise. Segmenting the market capitalization by time period in this manner allows for the relationship between market capitalization and private placement discounts to change from one time period to the next. That is, it allows us to test the hypothesis that the impact of market capitalization on private placement discounts has changed over time. If this were the case we would see different coefficients in each time period and/or only some of these variables would be statistically significant.

Similarly, holding other factors constant, larger discounts are generally observed for those companies with relatively high price-to-book ratios or with book values of less than zero. An abnormal price-to-book ratio (defined herein as any negative ratio or any ratio above 3.0) may imply greater uncertainty about a firm's valuation, a higher relative level of intangible versus tangible assets, more uncertainty about a firm's future prospects, and/or a firm that may be in financial distress. In assessing the impact of price-to-book ratios, we reviewed and tested the ratios across our data set and found it appropriate to assess this variable in segments. We assigned dummy variables to transactions that fell within certain specified ranges (0.0 to 1.25, 1.26 to 3.0). We note that the price-to-book ratio of the S&P 500 averaged 2.4 between 1970 and 2009, and most often ranged between 1.8 and 3.0. Transactions involving companies with price-to-book ratios within the ranges of 0.0 to 1.25 and 1.26 to 3.0, holding other factors constant, were consummated at significantly different discounts than other transactions. Price-to-book ratios of 0.0 to 1.25 tend to be associated with lower private placement discounts than are those with price-to-book ratios of 1.26 to 3.0. Price-to-book ratios outside the range of 0.0 to 3.0 tend to be associated with private placement discounts that are higher than those within that price-to-book range.

The transaction-specific variables we looked at concerned registration status, time period, and block size. Registration status is a dummy variable that is equal to 1 if the transaction was unregistered at the time of placement and not registered within one year of the transaction and equal to zero otherwise. We also set up dummy variables for time periods I through III (time period IV was the base case). These variables capture the effect of the changing holding period and related rules associated with changes in SEC Rule 144. Each time period variable was equal to 1 if the transaction took place in that particular time period, as defined earlier. In order to get a measure of block size, we used the natural log of quarters to sell under the 1% limit rule, as defined by SEC Rule 144.¹⁶

The contemporaneous market conditions we included were the natural log of the thirty-year U.S. Treasury yield and the twelve-month change in the S&P 500.

Our first dependent variable was the private placement discount (or premium), calculated based on the closing price the day before private placement was announced and the price at which the shares were placed, thus:

$$\text{Discount/Premium} = 1 - (\text{private placement price} / \text{closing price}).$$

We also estimated a model in which the log of the private placement price was the dependent variable. A purchaser of private placement shares would begin his analysis of how much he was willing to pay in terms of the current publicly traded price of the stock; hence, we began with the closing price on the day before the announcement of the private placement. For example, assume PublicCo is currently trading at \$20 per share and announces a private placement at \$18 per share (a 10% discount). A prospective investor in this private placement could be expected to examine PublicCo and determine whether he thought the PublicCo restricted stock was worth \$18 per share, given the restrictions as well as other pertinent factors. If so, he would likely purchase the private placement shares; if not, he would likely forego the purchase. The predicted discount is equal to 1 minus the predicted price divided by the closing price of the issuer's publicly traded stock. We note that the two dependent variables are algebraically equivalent.

The independent variables are summarized in Table 8.

Because the dependent variable and several of the independent variables in the second specification are in natural log form, the coefficients can be interpreted as elasticities. That is, a 1% change in the independent variable yields 1% multiplied by the given coefficient percent change in the dependent variable.

We defined the closing price as the closing price on the day before the earliest announcement of the private placement. We believe this is the best possible measure to determine a discount or premium that a prospective buyer would expect. Based on our analysis of private placements, we believe the coefficients on the independent variables would have the signs indicated in Table 9 (negative signs indicate that the variable leads to a lower predicted placement price and, thus, larger discounts; positive signs lead to smaller discounts).

The variables listed in Table 9 and used in our regression model are discussed below.

- A purchaser of private placement shares would certainly begin his analysis of how much he was willing to pay in terms of the current publicly traded price of the stock; thus, we believe the coefficient on log closing price will be positive and strongly statistically significant.

Table 8
 Variable Definitions

Variable	Definition
Quarters to Sell Under the 1% Limit Rule	Natural log of quarters to sell under the 1% limit rule, as outlined in Rule 144
Volatility	Natural log of issuer's one-year volatility
Price-to-Book Dummy Variable I	Dummy variable for which a value of 1 indicates the issuer's price-to-book ratio is greater than zero and less than or equal to 1.25, 0 if the issuer's price-to-book ratio is outside this range
Price-to-Book Dummy Variable II	Dummy variable for which a value of 1 indicates the issuer's price-to-book ratio is greater than 1.25 and less than or equal to 3.0, 0 if the issuer's price-to-book ratio is outside this range
S&P 500 Twelve-Month Change	Twelve-month change in the S&P 500 Index
Log Thirty-Year Treasury Yield	Natural log of U.S. thirty-year Treasury yield
Time Period I Dummy Variable	Dummy variable for which a value of 1 indicates transaction took place prior to 1990, 0 if transaction took place at another time
Time Period II Dummy Variable	Dummy variable for which a value of 1 indicates transaction took place from 1990 to 28 April 1997, 0 if transaction took place at another time
Time Period III Dummy Variable	Dummy variable for which a value of 1 indicates transaction took place from 29 April 1997 to 14 February 2008, 0 if transaction took place at another time
Log Market Capitalization Multiplied by Time Period I Dummy*	Natural log of market capitalization multiplied by 1 if the transaction took place in time period I (pre-1990), otherwise 0
Log Market Capitalization Multiplied by Time Period II Dummy*	Natural log of market capitalization multiplied by 1 if the transaction took place in time period II (1990 to 28 April 1997), otherwise 0
Log Market Capitalization Multiplied by Time Period III Dummy*	Natural log of market capitalization multiplied by 1 if the transaction took place in time period III (29 April 1997 to 14 February 2008), otherwise 0
Log Market Capitalization	Natural log of market capitalization main effect
Revenue	Issuer's previous year's revenue
Earnings	Issuer's previous year's earnings before tax

Table 9
 Expected Regression Coefficient Signs

Variable	Expected Sign
Log Closing Price	Positive
Log Quarters to Sell Under 1% Limit Rule	Negative
Registration Status	Negative
Time Period I Dummy	Negative
Time Period II Dummy	Negative
Time Period III Dummy	Negative
Log Thirty-Year Treasury	Unclear
S&P 500 Twelve-Month Change	Negative
Company Volatility	Negative
Log Market Cap (Main Effect)	Positive
Log Market Cap Period I	Positive
Log Market Cap Period II	Positive
Log Market Cap Period III	Positive
Price-to-Book Dummy Variables	Positive

- We would expect the sign on the natural log of quarters to sell under the 1% limit rule to be negative, indicating that as the value of this variable gets larger, we would expect the discount to also be larger. Or, in other words, as the size of the block being placed increases relative to the shares outstanding, we would expect a larger discount.
- For registration status, we also expect the sign to be negative, as shares that are unregistered (the dummy

variable is equal to 1) would expect to see larger discounts.

- We would expect all of the time period dummy variables to be negative. Our base case is time period IV, in which the restrictions are the most lax, as described earlier. Thus, we would expect, other things being equal, to see higher discounts in earlier time periods as a result of the more restrictive holding period rules under SEC Rule 144.
- It is not clear what the expected sign is for the natural log of thirty-year Treasury bond yields. If high bond yields encourage investors to invest in bonds instead of stocks, we would see larger discounts at times of high yields, and we would expect the sign to be negative. Alternatively, if high bond yields are associated with inflation, then equities may be more attractive, resulting in higher demand for private placements and driving discounts lower. In this case, the sign would be positive.
- We expect the sign on twelve-month change in the S&P 500 to be negative because during times of positive returns in the stock market we would expect investors to demand larger discounts to invest in restricted securities. (Put another way, why invest in restricted securities when investors can earn a good return in a very liquid market?)

- For the natural log of company volatility, we would expect the sign to be negative. We would expect to see larger discounts for companies that are more volatile.
- We would expect the sign of the natural log of market capitalization to be positive. This is because we would expect larger companies doing private placements to grant smaller discounts. For example, a large multinational industrial corporation would be able to place shares at a smaller discount than would a relatively new start-up company that had little or no revenue. As noted earlier, we divided this variable into its various time periods to allow for the relationship between market capitalization and observed discounts to vary based on which SEC Rule 144 regime was in place. We would expect the coefficient on each of the market capitalization by time period variables to be positive as well. Log market capitalization in time period IV is the base case.
- Finally, for price-to-book ratios, we looked at two dummy variables based on our data and historical trends in market price-to-book ratios. The first price-to-book dummy is equal to 1 for ratios in a range from 0.0 to 1.25, and it is zero otherwise. The second price-to-book dummy is equal to 1 for ratios in a range from 1.26 to 3.00 and is zero otherwise. We expect the sign on both of these to be positive, as companies with ratios in these ranges tend to be financially stronger overall and thus are able to do placements at smaller discounts.

Empirical Results

In this section we discuss the results from our main multiple regression model¹⁷ of the variables described above, as well as an expanded version that includes revenue, earnings, and stock market volatility (Table 10).

The results of the analysis were largely as we expected.¹⁸

- The coefficient on log of closing price was positive and strongly statistically significant, indicating that investors consider the current price of the stock in evaluating a private placement.
- The coefficient on log of quarters to sell under the 1% limit rule was negative and statistically significant, indicating that larger blocks as a percentage of shares outstanding would require larger discounts. This is consistent with the findings of earlier studies.
- The coefficient on registration status was negative and statistically significant, indicating that unregistered shares with no expectation of being registered in the near term sell at a discount to comparable

shares that have registration rights, agreements to register, or the expectation of registration.

- The time period I dummy variable was negative and statistically significant, indicating that transactions that took place during time period I, when the holding period requirements were most restrictive, generally took place at significant discounts compared to what was seen in later periods.
- Similarly, the coefficient on the dummy variable for time period II was also negative and statistically significant, though somewhat smaller in absolute magnitude. This indicates that relative to time period IV (the base case), the expected discount would be not as large in time period II as in time period I.
- The dummy variable for time period III was also negative; however, it was not statistically significant, indicating that the impact of the change in rules from time periods III to IV did not have a statistically significant impact. We note the relatively few observations from time period IV in our data set; thus, this effect may become more pronounced with the addition of more time period IV transactions.
- The coefficient on the log of thirty-year Treasury yields was positive and statistically significant. This indicates that in times of higher Treasury yields, investors may find private placements more attractive, thereby driving down discounts.
- The coefficient on the twelve-month change in the S&P 500 was negative and statistically significant. As expected, this indicates that during times of rising markets investors prefer the liquidity offered by readily marketable securities, thus requiring private placement discounts to widen to attract investors.
- The coefficient on the natural log of company volatility was negative and highly statistically significant, indicating that more volatile companies generally did private placements at higher discounts.
- The coefficient on the natural log of market capitalization (main-effect variable) was negative, which contradicted our expectations. Yet it was not statistically significant, indicating no relationship between market capitalization and private placement price in time period IV. However, the coefficients on the natural log of market capitalization in time periods I and II were positive, larger in absolute magnitude than the coefficient on the main effect, and statistically significant. This indicates that in periods I and II, larger companies did private placements at smaller discounts, which is in line with what we expected. The coefficient on the log of market capitalization in time period III was also positive, but it was not statistically significant. The total effect of market capitalization in a given time period is found

Table 10
 Regression Model Estimates (*t*-Statistics in Parentheses)

	Predicted	Private Placement Discount (PPD)	PPD-Expanded Model	Log Closing Price
Intercept		-0.164 (-2.17)**	-0.174 (-2.23)**	-0.195 (-1.83)
ln [†] (Closing Price)	POS	—	—	1.001*** (133.24)
ln (Quarters to Sell Under 1% Limit Rule)	NEG	-0.026*** (-5.44)	-0.026*** (-5.41)	-0.041*** (-5.84)
Registration Status	NEG	-0.056*** (-4.10)	-0.054*** (-3.95)	-0.093*** (-4.81)
Time Period I Dummy	NEG	-0.429*** (-4.42)	-0.469*** (-4.72)	-0.640*** (-4.67)
Time Period II Dummy	NEG	-0.359*** (-4.47)	-0.365*** (-4.27)	-0.517*** (-3.98)
Time Period III Dummy	NEG	-0.083 (-1.53)	-0.080 (-1.37)	-0.098 (-1.24)
ln (Thirty-Year Treasury)	UNC	0.092** (2.23)	0.093** (2.18)	0.125** (2.14)
S&P 500 Twelve-Month Change	NEG	-0.122*** (-4.88)	-0.113*** (-3.89)	-0.159** (-4.62)
ln (Company Volatility)	NEG	-0.084*** (-9.41)	-0.084*** (-8.82)	-0.133*** (-8.66)
ln (Market Cap [Main Effect])	POS	-0.011 (-1.15)	-0.011 (-1.12)	-0.015 (-1.24)
ln (Market Cap Period I)	POS	0.057*** (2.68)	0.067*** (3.11)	0.087*** (2.88)
ln (Market Cap Period II)	POS	0.061*** (3.85)	0.063*** (3.88)	0.092*** (3.43)
ln (Market Cap Period III)	POS	0.011 (1.05)	0.011 (1.02)	0.012 (0.97)
Price-to-Book Dummy (0 to 1.25)	POS	0.086*** (5.88)	0.085*** (5.82)	0.106*** (6.18)
Price-to-Book Dummy (1.26 to 3.00)	POS	0.029*** (3.39)	0.028*** (3.22)	0.042*** (3.72)
Revenue Previous Year		—	0.000 (0.26)	—
Earnings Before Tax Previous Year		—	-0.000 (-0.09)	—
S&P 500 Volatility		—	0.035 (0.43)	—
Number of Observations		1,863	1,851	1,863
Adjusted <i>R</i> ²		.2066	.2069	.9512
<i>F</i> -Statistic		27.03 <i>F</i> (14,1848)	22.09 <i>F</i> (17,1833)	2919.5 <i>F</i> (15,1847)
Prob > <i>F</i>		0.0000	0.0000	0.0000
Root MSE		0.1543	0.1536	0.2206

† ln = natural log; POS, positive; NEG, negative; UNC, unclear; MSE, mean squared error.

** Statistically significant at the 5% level.

*** Statistically significant at the 1% level.

by adding the coefficient on the main effect (−0.011) and the coefficient for the time period (Table 11).

- The coefficient on the price-to-book range of 0.0 to 1.25 dummy variable was positive and highly statistically significant, indicating that companies with ratios in this range tended to do placements at smaller discounts.

- The coefficient on the price-to-book range of 1.26 to 3.00 dummy variable was also positive and highly statistically significant, though substantially smaller in magnitude, indicating a smaller impact than that caused by the other price-to-book variable.
- Changing the dependent variable to log private placement price yielded substantively the same results.

Table 11
Marginal Impact of Market Capitalization by Time Period

Time Period	Main Effect	Period Effect	Total Impact	Statistically Significant?
I	-0.011	0.057	0.046	Yes
II	-0.011	0.061	0.050	Yes
III	-0.011	0.011	0.000	No
IV	-0.011	0.000	-0.011	No

Bold indicates Total Impact which is sum of main effect and period effect.

Each variable that was significant when discount was the dependent variable was significant in this specification as well. We are cognizant that the R^2 will be higher in this specification, as we are starting with the closing price of each transaction as one of the independent variables.

- Finally, in the expanded model we also looked at other variables, such as revenue, earnings, and stock market volatility. In each case the coefficients were not statistically significant.
 - In the cases of revenue and earnings we believe this indicates that their impact is already captured in the market capitalization of the company: that is, private placement investors are assuming that revenue and earnings are already accounted for in the market capitalization and thus consider other factors when determining whether and at what discount to purchase a private placement.
 - For S&P 500 volatility, some have argued that in times of high market volatility discounts should be higher because of the uncertainty. Alternatively, over the past thirty years high levels of market volatility have generally been associated with rising markets. The lack of statistical significance for this variable indicates that the two effects may be offsetting each other.

We believe our model helps explain variations in private placement discounts, as evidenced by the statistical significance of key variables. We note that this model can be used to provide a baseline estimate of the difference in marketability and liquidity between a freely tradable stock and an identical stock that is restricted. We believe that the predicted private placement discounts generated by the regression model above represent a minimum difference between freely tradable stock and restricted stock and that other factors that are non-quantifiable, such as transfer restrictions or limits on withdrawal, would make shares even less marketable and would increase the discount a buyer would demand to purchase such a security. Other factors that might reduce the DLOM include dividend policy and put rights.

Case Study

In this case study, we applied our regression-based model to the common equity of two companies, XYZ, Inc., a medical device company, and ABC, Inc., a heavy construction company. The two companies are identical except for their industries. Each company has an aggregate freely traded value of \$100 million and a book value of equity of \$50 million as of Fall 2009. The volatility of XYZ, Inc.'s common stock (as estimated through an analysis of guideline company volatilities) is 42.0%, while the volatility of ABC, Inc.'s common stock is 71%. Based upon these investment attributes, coupled with other variables discussed earlier, our model would indicate a predicted private placement discount for XYZ, Inc.'s common stock of roughly 29%, as shown in Table 12.¹⁹

Using the same basic setup, but changing the volatility assumption to 71% for ABC, Inc., results in a predicted discount of 34%. Thus, the more volatile nature of the construction industry resulted in a 400- to 500-basis point increase in the predicted private placement discount.

Volatility is not, however, the only metric that drives our client-specific private placement discount estimates. Market conditions prevailing on the valuation date also matter, as well as other company-specific and transaction-specific factors. Let's look again at XYZ, Inc., but let's change the valuation date to a period six months later, in 2010. The volatility of XYZ, Inc.'s common stock is now 24.0%, while the volatility of ABC, Inc.'s common stock is now 44%. Again assuming an aggregate freely traded value of \$100 million, the predicted private placement discount for XYZ, Inc., would be roughly 27%, as opposed to 29%. The lower discount reflects lower volatilities among the guideline companies as well as differences in other market and company-specific factors (on or about the valuation date). The predicted private placement discount for ABC, Inc., would be roughly 32%, as compared to 34% in the fall. Thus, changing market conditions reduced predicted private placement discounts by about 200 basis points over the six months in question. Table 13 summarizes these results.

Table 12
 Case Study

	Coefficient	Fall 2009 XYZ, Inc.		
		Base Value	Input	Contribution
Log Thirty-Year Treasury, %	0.092	4.23	1.44	0.13
Log Quarters to Sell Under 1% Limit Rule	-0.026	22.00	3.09	(0.08)
Log Company Volatility	-0.084	0.42	(0.87)	0.07
Registration Status	-0.056	1.00	1.00	(0.06)
S&P 500 Twelve-Month Change, %	-0.122	7.00	0.07	(0.01)
Time Period I Dummy	-0.429	1.00	1.00	(0.43)
Time Period II Dummy	-0.359	—	—	—
Time Period III Dummy	-0.083	—	—	—
Log Market Cap Main Effect, \$MM	-0.011	100.00	4.61	(0.05)
Log Market Cap Period I, \$MM	0.057	100.00	4.61	0.26
Log Market Cap Period II, \$MM	0.061	—	—	—
Log Market Cap Period III, \$MM	0.011	—	—	—
Price-to-Book Dummy (0.0 to 1.25 = 1)	0.086	—	—	—
Price-to-Book Dummy (1.26 to 3.00 = 1)	0.029	1.00	1.00	0.03
Intercept	-0.164	1.00	1.00	(0.16)
Predicted (Discount)/Premium				-29.1%

Table 13
 Case Study Results

Valuation Date	XYZ, Inc., Healthcare	ABC, Inc., Construction
Fall 2009, % (% Volatility)	29 (42)	34 (71)
Spring 2010, % (% Volatility)	27 (24)	32 (44)

Conclusions

Our study has shown the effect of a variety of factors on private placement discounts and has discussed how these predicted discounts can be used as a basis for determining DLOMs.

Our model showed that changes in SEC Rule 144 holding periods have had a significant impact on private placement discounts. We believe our study represents the first attempt to capture the impact of the changing SEC Rule 144 holding period requirements while simultaneously holding constant other explanatory variables. Not surprisingly, our analysis showed the sharp decrease in magnitude of discounts resulting from changes in Rule 144. We also found that the placement of unregistered shares without the prospect of being registered in the near term resulted in larger private placement discounts.

We captured the impact of market conditions on private placement discounts, as measured by the yield on thirty-year Treasury bonds (higher bond yields lead to lower discounts) and the performance of the S&P 500 (stronger S&P 500 performance in the prior year leads to higher discounts). These factors have not been included in previous analyses, and we believe the addition of contemporaneous market conditions provides better estimates than those generated by previous models.

We also noted that the relationship between market capitalization and private placement discounts has varied over time. Specifically, prior to 1997 (that is, during periods I and II), there was a significant relationship between size and private placement discounts, as larger companies were able to do private placements at smaller discounts. Smaller companies are also more often in a position in which the private placement was a funding that was, if not a last resort, of vital importance to the company's ability to continue as a going concern. After 1997, size seems to have had less of an impact on private placement discounts.

We found that the volatility of a company's stock price was a significant factor in predicting private placement discounts: as a company's volatility increased, the observed private placement discount got larger.

We found that the following factors did not impact private placement discounts, revenue, earnings, stock market volatility, and industry. Finally, we discussed how a private placement discount could be converted to an estimated DLOM through the analysis of qualitative factors such as dividend policy and transfer restrictions, among others.

We will continue to update our database of transactions and models going forward, which will allow us to trace the evolution of the impact of the factors discussed over time.

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Endnotes

1. When referring to the discount for lack of marketability (DLOM), we are actually including the concepts of lack of liquidity and lack of marketability.
2. Thomson Financial for the period 2000 to 2008 and Capital IQ for the period 1 January 2009 to 30 September 2009. Data on each private placement transaction were gathered in the same manner. Number of shares placed and private placement price were verified in both press releases and SEC filings. Changes in registration status were verified in SEC filings.
3. We made this elimination because we believe these shares to be substantially more speculative than shares of higher priced companies.
4. We looked at the closing price the day before the announcement in order to get the best measure of the discount. Purchasers of the private placement shares base their analyses on the then-current price; thus, we sought to compare the price they paid for the private placement shares with the market price of those shares before any announcement of a private placement transaction.
5. In January 1972, the SEC adopted Rule 144 as an objective safe harbor for the resale of restricted securities. In order to prevent a seller from purchasing unregistered securities with a view toward resale, Rule 144 required a minimum holding period of two years. In 1990 (the beginning of time period II), Rule 144 was amended to permit the purchaser of restricted stock to “tack” on to the holding period of the previous owner of these securities in certain transactions. In layman’s terms, the 1990 amendment increased the liquidity and marketability of restricted stock by allowing the subsequent private purchasers of such shares from a non-affiliate to sell the securities in the public market sooner than they could have prior to the 1990 amendment. On 29 April 1997 (the beginning of time period III), Rule 144 was again amended to enhance the liquidity and marketability of restricted stock by shortening the initial holding period from two years to one year. It is important to note, however, that the median discount for one of the studies of time period III in the above tabulation is reduced by the inclusion of transactions with certain registration rights sometimes offered to investors in unregistered shares. In our opinion, the existence of these rights would likely substantially increase the perceived liquidity and marketability of the blocks purchased. Finally, in February 2008 (the beginning of time period IV), Rule 144 was again amended to enhance the liquidity and marketability of restricted stock by shortening the initial holding period from one year to six months.
6. t statistic was 5.285, with 696 degrees of freedom. The associated p -value was less than .0001.
7. Observations were dropped from our data set only if we could not verify the private placement price or number of shares placed or if the stock traded less than 1,000 shares the week prior to the private placement announcement. Stocks with closing prices below \$1 were also omitted. In cases in which the only missing piece of data was company volatility we employed multiple imputation to estimate these values. This occurred in less than 1% of the observations.
8. This extremely large transaction was a private placement done by Merrill Lynch & Company in late 2007, as the financial crisis was beginning to impact the major investment banks. Transactions of this magnitude were rare. There were five transactions (out of 1,863) that were larger than \$1 billion.
9. Karen Wruck, “Equity Ownership Concentration and Firm Value: Evidence from Private Equity Financings,” *Journal of Financial Economics* 23 (1989): 3–28.
10. Michael Hertzel and Richard L. Smith, “Market Discounts and Shareholder Gains for Placing Equity Privately,” *The Journal of Finance*, 48 (1993): 459–485.
11. William L. Silber, “Discounts on Restricted Stock: The Impact of Illiquidity on Stock Prices,” *Financial Analysts Journal* Vol 47, No. 4, (1991):60–64.
12. Mukesh Bajaj, David J. Denis, Stephen P. Ferris, and Atulya Serrin (referred to as Bajaj), “Firm Value and Marketability Discounts,” *Journal of Corporation Law* 27 (2001):89–115.
13. Mark Mitchell and Mary Norwalk, “Assessing and Monitoring Bajaj: Debunking the Reliability of Marketability Discount Studies and the 7.23% Solution,” *Business Valuation Review* (Spring 2008): 3–17.
14. Several authors measured the discount ten days after the announcement.
15. In this article, any references to “log” refer to the natural log.
16. Quarters to sell under the 1% limit rule is defined as number of shares placed divided by 1% of shares outstanding, a variant of block to shares outstanding. An alternative would have been to look at the volume limit portion of Rule 144, which limits sales to the average weekly reported trading volume during the

four weeks preceding such sale. We found the data on quarters to sell to be more reliable than volume data; thus, we relied on it rather than on volume data.

17. We corrected for heteroscedasticity by employing robust standard errors in our estimation of the model. We also imputed missing values in the data set using standard imputation procedures. Less than 1% of the data were imputed.
18. We also estimated a model with a panel of dummy variables based on Standard Industrial Classification (SIC) code to estimate the impact of a company's industry on the private placement discount. We found

that there was no statistically significant effect of industry on private placement discounts.

19. We assume that Time Period I to capture the more restrictive holding period requirements of that time are closest to owning shares in a privately held company for which there is no set timetable for liquidity and whose holding period may be much longer than the two years required under Rule 144. Further, we would typically look at a range of volatility and holding period assumptions (as measured by the quarters to sell variable) to establish a range of results for sensitivity analysis purposes.